REVIEW

1) The probability that the surprise toy is in container A or container B is given by the ratio of the volumes of type A and B containers and the total volume of all of the containers.

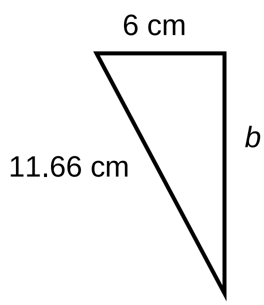
Volume of the cylinder:

$V = \pi r^2 h$

$V = \pi \times 4^2 \times 6$

$V = 96\pi \text{ cm}^3 = 301.59 \text{ cm}^3$

Volume of the cone:



Height of the cone using the Pythagorean Theorem.

$c^2 = a^2 + b^2$

$b = \sqrt{(11.66)^2 - (6)^2}$

$b = 10$

Volume of the cone

$V = \dfrac{\pi r^2 h}{3}$

$V = \dfrac{\pi 6^2 \times 10}{3}$

$V = \dfrac{\pi \times 36 \times 10}{3}$

$V = 120\pi \text{ cm}^3 = 376.99 \text{ cm}^3$

Volume of the half sphere:

$V = \dfrac{1}{2} \times \dfrac{4\pi r^3}{3}$

$V = \dfrac{1}{2} \times \dfrac{4\pi 3^3}{3}$

$V = 18\pi \text{ cm}^3 = 56.49 \text{ cm}^3$

Volume of the cube:

$V = c^3$

$V = 6^3$

$V = 216 \text{ cm}^3$

Probability calculation:

$$P(\text{A or B}) = \frac{\text{volume container A} + \text{volume container B}}{\text{total volume}}$$

$$P(\text{A or B}) = \frac{(40 \times 301.59) + (52 \times 376.99)}{(40 \times 301.59) + (52 \times 376.99) + (30 \times 56.49) + (24 \times 216)}$$

$$P(\text{A or B}) = \frac{31\,667.25}{38\,547.71}$$

$$P(\text{A or B}) = \frac{82.2}{100}$$

Answer: Given the volume of each container, the probability that the surprise toy will be in a type A or type B container is **82.2%.**

REVIEW

$$\frac{\left(\left(a^3 \bullet b^5\right)^8\right)^{\frac{1}{2}} \bullet \left(a^2 \bullet c^3\right)^5 \bullet \left(c^5 \bullet b\right)^{-2} \bullet a^3}{\left(a^3 \bullet c^{-1}\right)^{-5} \bullet \left(b^2\right)^{-7} \bullet \left(\left(b^4 \bullet a^5\right)^{24}\right)^{\frac{1}{3}}} = 1$$

$$\frac{a^{12} \bullet b^{20} \bullet a^{10} \bullet c^{15} \bullet c^{-10} \bullet b^{-2} \bullet a^3}{a^{-15} \bullet c^5 \bullet b^{-14} \bullet b^{32} \bullet a^{40}} = 1$$

2)

$$\frac{a^{25} \bullet b^{18} \bullet c^5}{a^{25} \bullet c^5 \bullet b^{18}} = 1$$

$$\frac{a^{25} \bullet b^{18} \bullet c^5}{a^{25} \bullet b^{18} \bullet c^5} = 1$$

$$1 = 1$$

3) **Use Pythagorean Theorem to find the height of the pyramid**

- Leg of the right-triangle

  300 ÷ 2 = 150 cm

- Height of the pyramid

  $\sqrt{250^2 - 150^2} = 200\,\text{cm}$

Calculate the volume of the pyramid

$$V = \frac{A_b \times H}{3}$$

$$A_b = 300^2 = 90\ 000$$

$$V = \frac{90\ 000 \times 200}{3} = 6\ 000\ 000\ \text{cm}^3$$

Convert units of volume and capacity

- Converting from centimetres to metres 6000 000 ÷ 1000 ÷ 1000 = 6 m^3

One pack contains 1 m^3 of sand. Samuel will need 6 packs of sand.

Answer: Samuel will need **6** packs of sand to build his sand castle.

REVIEW

4) **Total surface area to paint:**

Square–base + cylindrical pole + lamp shade

Square-base prism:

Lateral area of base + Area of one square base – Area of cylindrical base

Lateral area of base:

$$\begin{aligned} LA &= Pb \times h \\ &= (14 \times 4) \times 1 \\ &= 56 \text{ cm}^2 \end{aligned}$$

Area of one square base:

$$\begin{aligned} A_b &= L \times W \\ A_b &= 14 \times 14 \\ A_b &= 196 \text{ cm}^2 \end{aligned}$$

Area of cylindrical base:

$$\begin{aligned} A_{cylindricalbase} &= \pi r^2 \\ A_{cylindricalbase} &= (3.14)(1.5)^2 \\ A_{cylindricalbase} &= 7.07 \text{ cm}^2 \end{aligned}$$

Total area of square-base prism: 56 cm^2 + 196 cm^2 – 7.07 cm^2 = 244.93 cm^2

Cylindrical Pole:

Lateral area of cylinder:

$$\begin{aligned} LA &= Pb \times h \\ LA &= 2\pi rh \\ LA &= 2(3.14)(1.5)(21) \\ LA &= 197.92 \text{ cm}^2 \end{aligned}$$

Hemispherical Lamp Shade:

Lateral Area of hemisphere:

$$\begin{aligned} LA &= 2\pi r^2 \\ LA &= 2(3.14)(11)^2 \\ LA &= 760.27 \text{ cm}^2 \end{aligned}$$

Total surface area to paint: 244.93 + 197.92 + 760.27 = 1203.12 cm^2

Answer: The total surface area that Shelly will be painting is **1203.12 cm^2.**

REVIEW

5)

$$A = \frac{14x^4 - 10x^3 + 8x}{2x}$$
$$A = 7x^3 - 5x^2 + 4$$

$$B = (2x^2 - 3x)(4x + 1)$$
$$B = 8x^3 - 10x^2 - 3x$$

$$C = 13x^3 - 9x^2 + 7x - 2 + 2x^2 - 3x - 6$$
$$C = 13x^3 - 7x^2 + 4x - 8$$

$$D = (4x^3 + x^2 - 9x) - (3x^3 - 5x^2 - 1)$$
$$D = x^3 + 6x^2 - 9x + 1$$

Algebraic expression $A + 2B - 3C + D$

$$A = 7x^3 - 5x^2 + 4$$

$$2B = 2(8x^3 - 10x^2 - 3x) = 16x^3 - 20x^2 - 6x$$

$$-3C = -3(13x^3 - 7x^2 + 4x - 8) = -39x^3 + 21x^2 - 12x + 24$$

$$D = x^3 + 6x^2 - 9x + 1$$

$$\begin{aligned} A + 2B - 3C + D = {} & 7x^3 - 5x^2 + 0x + 4 \\ & 16x^3 - 20x^2 - 6x + 0 \\ & -39x^3 + 21x^2 - 12x + 24 \\ & 1x^3 + 6x^2 - 9x + 1 \\ \hline & -15x^3 + 2x^2 - 27x + 29 \end{aligned}$$

Answer: The simplified expression of $A + 2B - 3C + D$ is $\boldsymbol{-15x^3 + 2x^2 - 27x + 29}$**.**

REVIEW

6) Volume of the cylinder:

$$V = \pi r^2 h$$
$$V = \pi(4)^2 \bullet 10$$
$$V = 502.65 \text{ cm}^3$$

Volume of the cylinder = Volume of the cone

Height of the cone

$$502.65 = \frac{1}{3}\pi r^2 h$$
$$1507.95 = \pi r^2 h$$
$$1507.95 = \pi(5^2)h$$
$$\frac{1507.95}{\pi(5^2)} = h_{\text{cone}}$$
$$\frac{1507.95}{78.54} = h_{\text{cone}}$$
$$19.2 \text{ cm} = h_{\text{cone}}$$

Total height of the sculpture: $10 + 19.2 = 29.2$ cm

Total volume of the sculpture:

$$502.65 \times 2 = 1005.3 \text{ cm}^3$$

$$1005.3 \text{ cm}^3 = 1.0053 \text{ L}$$

Capacity mould < 1.25 L Hydro-Rock

Answer: **Yes, Tommy is correct.**

Tommy's sculpture will fit into the display case. The capacity of his mould does not exceed 1.25 L and its height is less than 30 cm.

REVIEW

7) **Simplifies the algebraic expression representing the distance between the dock and Heron Island.**

$$m\,\overline{QH} = \frac{4x^2 - 6x}{2x}$$

$$m\,\overline{QH} = \frac{4x^2}{2x} - \frac{6x}{2x}$$

$$m\,\overline{QH} = 2x - 3$$

Calculates the value of *x*

$$m\,\overline{QH} + m\,\overline{HS} = 9$$

$$(2x - 3) + (3x - 8) = 9$$

$$5x - 11 = 9$$

$$5x = 9 + 11$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4$$

Calculates the distance between the dock and Heron Island, $m\,\overline{QH}$

$$m\,\overline{QH} = 2x - 3$$

$$m\,\overline{QH} = 2(4) - 3$$

$$m\,\overline{QH} = 8 - 3$$

$$m\,\overline{QH} = 5$$

Calculates the distance between Heron Island and Mermaid Island, $m\,\overline{HS}$

$$m\,\overline{HS} = 3x - 8$$

$$m\,\overline{HS} = 3(4) - 8$$

$$m\,\overline{HS} = 12 - 8$$

$$m\,\overline{HS} = 4$$

Calculates the shortest distance between Mermaid Island and the dock, $m\,\overline{SQ}$

$$m\,\overline{QH}^2 + m\,\overline{HS}^2 = m\,\overline{SQ}^2$$

$$\sqrt{m\,\overline{QH}^2 + m\,\overline{HS}^2} = m\,\overline{SQ}$$

$$\sqrt{5^2 + 4^2} = m\,\overline{SQ}$$

$$\sqrt{25 + 16} = m\,\overline{SQ}$$

$$\sqrt{41} = m\,\overline{SQ}$$

$$m\,\overline{SQ} \approx 6.4\text{km}$$

Answer: They travel **6.4 km** to return to the dock.

REVIEW

8) Determine the volume of one wax block:

10 cm × 15 cm × 5 cm = 750 cm^3

5 blocks of wax: 3750 cm^3

Determine the volume of the cylindrical candles:

Cylinder: area × height → $\pi(4)^2(6) \approx 301.593\,\text{cm}^3$

Cone: height → Pythagoras:

$$5^2 = 4^2 + h^2$$
$$25 - 16 = h^2$$
$$3 = h$$

Cone: area × h ÷ 3 → $\frac{\pi(4)^2 \times 3}{3} \approx 50.265\ \text{cm}^3$

Combined volume: $301.593 + 50.265 = 351.858\ \text{cm}^3$

Volume of 3 candles: 351.858 × 3 = 1055.574 cm^3

Determine the remaining wax:

3750 cm^3 – 1055.574 cm^3 = 2694.426 cm^3

Determine the volume of wax needed for a spherical candle with a radius of 4 cm:

Volume of sphere: $\frac{4}{3}\pi(4)^3 = \frac{256\pi}{3} \approx 268.082\ \text{cm}^3$

Determine the quantity of spherical candles that can be made from the leftover wax:

$2694.426 \div 268.082 = 10.050...$

10 candles

Answer: Kelly can make **10** spherical candles.

REVIEW

9) RATIO OF THE VOLUMES

$$k^2 = \frac{468 \text{ cm}^2}{208 \text{ cm}^2}$$
$$k^2 = 2.25$$
$$k = \sqrt{2.25} = 1.5$$
$$k^3 = 1.5^3 = 3.375$$

VOLUME OF CURRENT POT

$$V_1 = A_b \times h$$
$$V_1 = 208 \text{ cm}^2 \times 20 \text{ cm}$$
$$= 4160 \text{ cm}^3$$

VOLUME OF NEW POT

$$V_2 = 4160 \text{ cm}^3 \times k^3$$
$$V_2 = 4160 \text{ cm}^3 \times 3.375$$
$$V_2 = 14\,040 \text{ cm}^3$$

NUMBER OF JARS USING CURRENT POT

$$640 \text{ mL} = 640 \text{ cm}^3$$
$$\text{Number of jars} = \frac{4160 \text{ cm}^3}{640 \text{ cm}^3}$$
$$= 6.5 \text{ jars} \rightarrow 6 \text{ full jars}$$

NUMBER OF JARS USING NEW POT

$$\text{Number of jars} = \frac{14\,040 \text{ cm}^3}{640 \text{ cm}^3}$$
$$= 21.9375 \text{ jars} \rightarrow 21 \text{ full jars}$$

DIFFERENCE IN NUMBER OF FULL JARS

21 – 6 = 15 more full jars

CONCLUSION: Ms. Greco will be able to fill **15** more full jars of tomato sauce using her new pot compared to her current pot.

REVIEW

10)

DIMENSIONS OF RECTANGLE

Height of the rectangle

Greatest common factor of the area is $16x$.

$\therefore$ The height of the rectangle is ($16x$) cm.

Base of the rectangle

$$\frac{(96x^3 + 64x^2 + 80x)}{16x} = 6x^2 + 4x + 5$$

The base of the rectangle is: $(6x^2 + 4x + 5)$ cm.

DIMENSIONS OF THE TRIANGLE

Height of the triangle

$16x - (4x + 2x) = 10x$

The height of the triangle is: ($10x$) cm.

Base of the triangle

$(6x^2 + 4x + 5) - (4x + 6 + 4x + 6)$

$(6x^2 + 4x + 5) - (8x + 12)$

$6x^2 + 4x + 5 - 8x - 12$

$6x^2 - 4x - 7$

The base of the triangle is: $(6x^2 - 4x - 7)$ cm.

AREA OF THE TRIANGLE

$$A = \frac{10x(6x^2 - 4x - 7)}{2}$$
$$= \frac{60x^3 - 40x^2 - 70x}{2}$$
$$= 30x^3 - 20x^2 - 35x$$

The area of the triangle is: $(30x^3 - 20x^2 - 35x)\text{cm}^2$.

AREA OF THE BLUE SECTION

$(96x^3 + 64x^2 + 80x) - (30x^3 - 20x^2 - 35x)$

$96x^3 + 64x^2 + 80x - 30x^3 + 20x^2 + 35x$

$(66x^3 + 84x^2 + 115x)$ cm^2

CONCLUSION

The simplified algebraic expression that represents the area of the blue section of the flag is (**$66x^3 + 84x^2 + 115x$**) cm^2.