ANALYTIC GEOMETRY

Distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$
how far, length

$$\sqrt{(x2-x1)+(y2-y1)}$$

Midpoint between points $A(x_1, y_1)$ and $B(x_2, y_2)$
halfway, middle, divided into equal parts

$$\frac{x2+x1}{2}, \frac{y2+y1}{2}$$

Point of Division: fraction $\frac{a}{b}$ of the way from $A(x_1, y_1)$ to $B(x_2, y_2)$ STARTING FROM **A**!

part way, division of a line
if given a ratio p:p → convert to fraction $\frac{part}{whole}$

$$x1+\frac{a}{b}(x2-x1), y1+\frac{a}{b}(y2-y1)$$

EQUATION OF A LINE

Functional form
Equation (rule): **y = <u>a</u>x + <u>b</u>**

Slope, rate of change — y-intercept, initial value

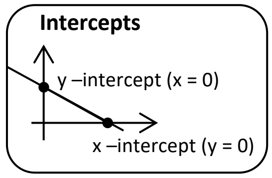
a = $\frac{y2-y1}{x2-x1}$ **b = y_1 – a(x_1)**

i.e.: A(6, 7) and B(15, 25)

1) $\frac{25-7}{15-6}=\frac{18}{9}=2$
[y = 2x + b]

2) **7** = 2(**6**) + b
7 = 12 + b
7-12 = b
b = **-5**

3) **y = 2x -5**



General form **ax + by + c = 0**
*convert to Functional form by isolating 'y'
Ex. 3x + 2y - 6 = 0
2y = **-3x + 6** divide by **2**
y = **-1.5x + 3**

Parallel & Perpendicular LINES

<u>Parallel lines</u>: have the same slope (a)
Find the equation **<u>parallel</u>** to y=2x + 8
And goes through the point (5, 25)

1) Find a: 2→2 (same slope)
2) Find b: **25** = 2 (**5**) + b
25 = 10 + b
25 - 10 = b →b=15
3) Write the rule: y = 2x +15

<u>Perpendicular lines</u>: negative reciprocal slopes - flip the fraction *<u>change</u>* the sign
Find the equation **<u>perpendicular</u>** to y=3x + 5
And goes through the point (9, 15)

1) Find a: 3→-1/3
2) Find b: **15 =** -1/3 (**9**) + b
15 = -3 + b
15 + 3 = b → b = 18
3) Write the rule y = -1/3(x) + 18

<u>Coincident Lines:</u> SAME slope & b-value
<u>Distinct Lines</u>: DIFFERENT slope & b-value

<u>COMPARISON METHOD</u>

finding a point of intersection with 2 functional form equations

y = 2x + 7
y = -x -5

2x + 7 = -x -5
2x **+ x** = -5 **-7**
3x = -12
x = -12 ÷ **3** = -4

y = 2(**-4**) + 7 = -1

(-4, -1)

<u>ELIMINATION METHOD</u>

Finding the value of x and y with 2 general form equations

3 × (2x + 5y = -4**)**
2 × (3x – 2y = 13**)**

$\cancel{6x}$ + 15y = -12
– $\cancel{6x}$ – 4y = 26

19y = -38
y = -38 **÷ 19** = -2

2x + 5**(-2)** = -4
2x = -4 **+10**
x = 6 **÷ 2** = 3

(3, -2)

<u>Substitution Method</u>

Finding the solution or point of intersection when One equation has x or y isolated

y= $\boxed{4x + 5}$ — Sub (4x+5) into (y)

3x + 2$\boxed{y}$ = 43

3x + **2(4x+5)** = 43
3x + 8x + 10 = 43
11x + 10 = 43
11x = 43-10
11x = 33 ÷ 11
x = 3

y = 4(3) + 5
y = 12 + 5 = 17
Solution Set (3, 17)

Strategies to Find a Missing Point (x,y)

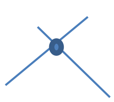
<u>Midpoint</u>
If it's halfway, in the middle, in the center

<u>Point of Division</u>
if fraction (leave a fraction)
if ratio (transform into a fraction)

$$part:part \rightarrow \frac{part}{whole}$$

<u>Systems of Linear Relation</u>

- **Comparison**
- **Substitution**
- **Elimination**



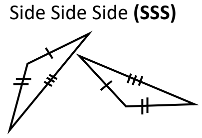
To find the intersection or corner where 2 lines meet.

Note: you may have to find the equation of a line
y = ax+b
Find a (slope) and b (y-intercept)

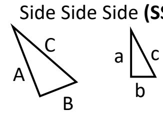
Congruent Triangles
Exactly the same

Similar Triangles
Larger/smaller by scale factor k

Side Side Side (**SSS**)

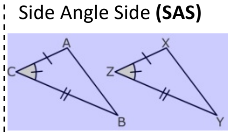


Side Side Side (**SSS**)

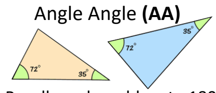


$\frac{A}{a}=\frac{B}{b}=\frac{C}{c}=$ **k**

Side Angle Side (**SAS**)

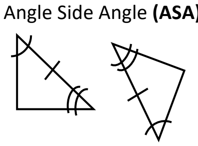


Angle Angle (**AA**)

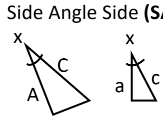


Recall: angles add up to 180⁰!

Angle Side Angle (**ASA**)



Side Angle Side (**SAS**)



$\frac{A}{a}=\frac{C}{c}=$ **k** < x = < x

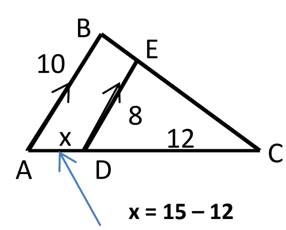
MISSING MEASURES IN TRIANGLES

Similar triangles: Draw two triangles, set up ratio and cross multiply

$$\frac{AB}{DE}=\frac{AC}{DC}$$

$$\frac{10}{8}=\frac{AC}{12}$$

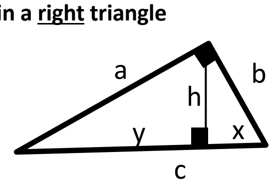
AC = 15



x = 15 – 12
x = 3

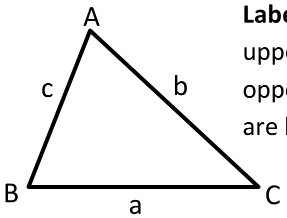
Metric relations in a <u>right</u> triangle

$a^2 = y*c$
$b^2 = x*c$
$h^2 = y*x$
$a*b = h*c$



Don't forget **Pythagoras!**
$\mathbf{a^2 + b^2 = c^2}$

SINE LAW: for all triangles



Labels: angles are upper case (A) sides opposite to angles are lower case (a)

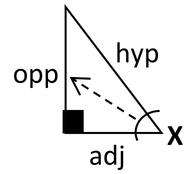
$$\boxed{\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}}$$

→ Fill in formula, remove extra ratio, cross multiply

remember

- if your unknown is an **angle**, use sin^{-1}
- if your angle SHOULD be obtuse (> 90°)… 180° - answer

TRIGONOMETRY: for RIGHT triangles!



sin**X** = $\frac{o}{h}$

cos**X** = $\frac{a}{h}$

tan**X** = $\frac{o}{a}$

To determine measure of angle **X**, use inverse function

sin^{-1} cos^{-1} tan^{-1}

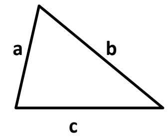
remember

- your calculator must be in DEGree mode
- keep 4 decimal places in your ratio

AREA OF TRIANGLES

1. $\frac{\boldsymbol{base \times height}}{\mathbf{2}}$ — *use this when the base is perpendicular to the height (look for right angle!)

2. **HERO'S formula:** use when you know all 3 sides



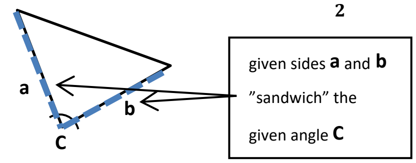
calculate semi-perimeter p = $\frac{a+b+c}{2}$

Area = $\sqrt{p(p-a)(p-b)(p-c)}$

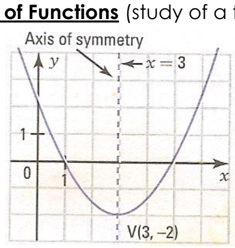
BEDMAS

3. **Trigonometric (sandwich) formula:** use when you know 2 sides and one angle (you might need SINE LAW)

Area = $\frac{\boldsymbol{a \times b \times sinC}}{\mathbf{2}}$

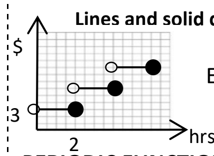


PROPERTIES of Functions (study of a function)



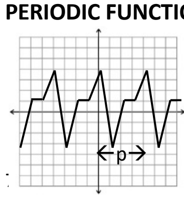
Domain (All x-values)
How far left, How far right : **]-∞, +∞[or IR**
Range (All y-values)
How far down, How far up : **[-2, +∞[**
Intercepts (where the curve touches an axis)
Zeros or x-intercepts (x,0) : **{1, 5}**
Initial Value or y-intercept (0,y): **{2}**
Variation (x-values)
Increasing *(going up)* : **[3, +∞[**
Decreasing *(going down)* : **]-∞,3]**
Sign (x-values)
Positive*(curve is above x-axis)***:]-∞,1]U[5, +∞[**
Negative*(curve is below x-axis)***: [1,5]**
Extrema (y-values) → *if infinite, no extreme*
Max = None Min = -2

STEP FUNCTION – open circles are **not** included
Lines and solid dots are included



Ex. Pay $3 for every 2 hours in a parking lot

PERIODIC FUNCTION – repeating pattern / wave



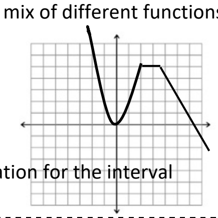
Period **(p)** = the length of one repeating pattern

To find a y value: f(44) with a period of 7

$\frac{44}{7} = 5\frac{2}{7}$ so count **2** units from the beginning of cycle to find y. If f(-44) count to left from end of cycle.

PIECEWISE FUNCTION – mix of different functions

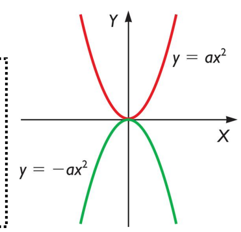
$$f(x) = \begin{cases} \mathbf{x^2} & x \le 5 \\ \mathbf{25} & 5 < x \le 10 \\ \mathbf{-2x+45} & x > 10 \end{cases}$$



- Choose interval of x
- Plug **given** value into equation for the interval
- Solve for **missing** value

QUADRATIC FUNCTION
(second-degree polynomial function)
f(x) = ax²

F(x) = y value
f(#) means replace the x with # solve for y



0 < a < 1 wider (a is a fraction or decimal)
a > 1 thinner

To determine "a", plug in one point on curve
Ex. Point (3, 27) → $y = ax^2$
$27 = a(3)^2$
**BEDMAS $27 = a(9)$ divide by 9
$3 = a$ → $\mathbf{y = 3x^2}$