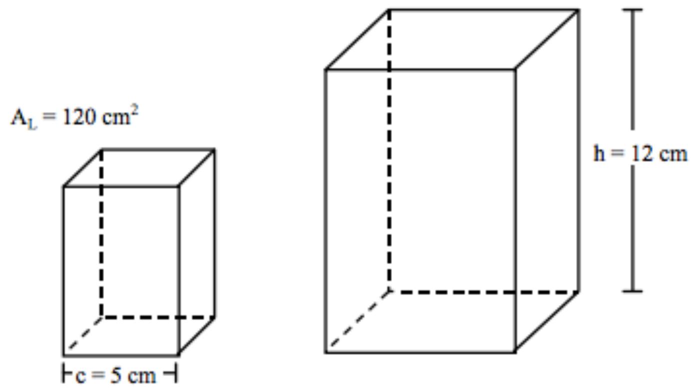
Week of April 20th
The Answer Key is at the bottom of the document
Please try doing each question before looking at the answer key!

Question 1

The two right prisms with square bases represented below are similar.



The lateral area of the smaller prism is 120 cm^2 and one side of its base measures 5 cm.
If the height of the larger prism is 12 cm, what is its volume?

Show all your work.

Answer The volume of the larger prism is _______________ cm^3.

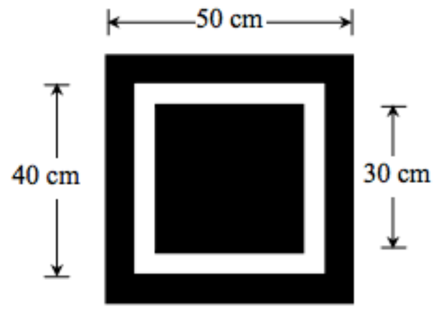
Question 2

2 The diagram on the right shows a square target. Its sides measure 50 cm each.

The design used on this target was created by drawing two squares whose sides measure 40 cm and 30 cm respectively.

A section of this target is white.

Someone throws a dart at this target.



What is the probability that the dart will land in the white section of the target?

A) $\frac{1}{6}$

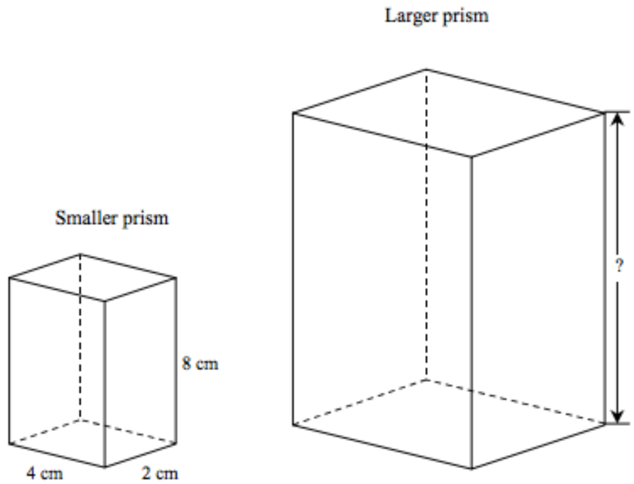
B) $\frac{7}{25}$

C) $\frac{1}{3}$

D) $\frac{16}{25}$

Question 3

3 The two rectangular right prisms shown below are similar. The dimensions of the smaller prism are given in the diagram below. The volume of the larger prism is 1728 cm^3.



The prisms are **not** drawn to scale.

What is the height of the larger prism?

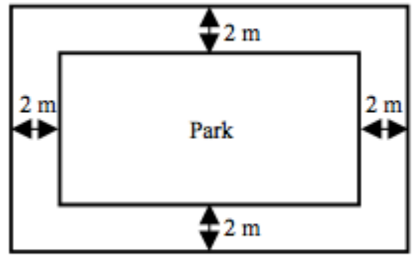
Show all your work.

Answer: The height of the larger prism is ______________ cm.

Question 4

The length of a rectangular park is 2 times greater than its width.

The park is framed by sidewalk that is 2 m wide, as shown on the right.



The area of the park is increased by 136 m^2 if the sidewalk is included.
What is the area of the park without the surrounding sidewalk?

Work

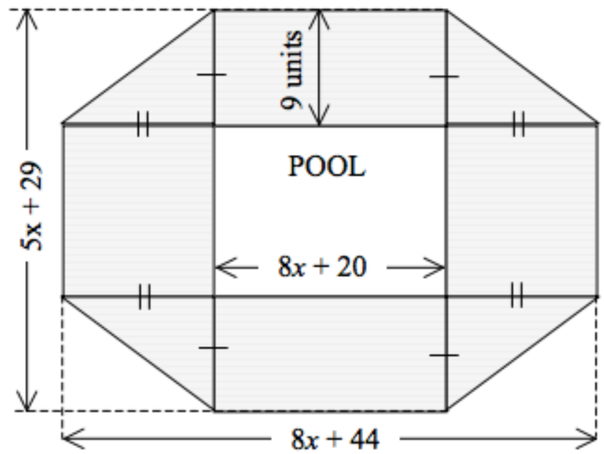
Result : The area of the park without the sidewalk is ______________ m^2.

Question 5

The floor of a wooden deck is designed around a rectangular pool as shown. The deck is the shaded area.

What is the simplified algebraic expression for the length of fencing that will be required around the outer perimeter of the deck?

Show all your work.



Show all your work.

Answer: The simplified algebraic expression for the length of the fence is

____ units.

Question 6

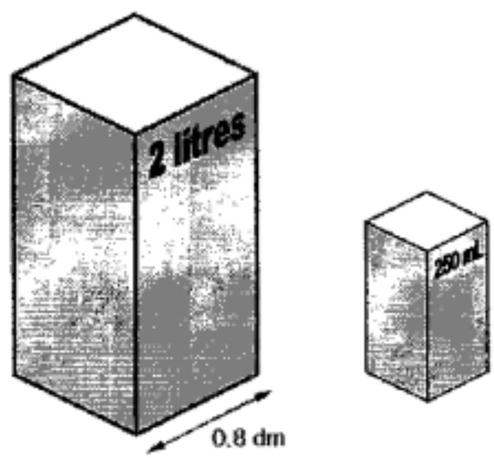
Using the laws of exponents, show that the following expression is true.

$$\frac{\left(\left(a^{3} \bullet b^{5}\right)^{8}\right)^{\frac{1}{2}} \bullet \left(a^{2} \bullet c^{3}\right)^{5} \bullet \left(c^{5} \bullet b\right)^{-2} \bullet a^{3}}{\left(a^{3} \bullet c^{-1}\right)^{-5} \bullet \left(b^{2}\right)^{-7} \bullet \left(\left(b^{4} \bullet a^{5}\right)^{24}\right)^{\frac{1}{3}}} = 1$$

Question 7

The *Garden of Eden* juice company is launching a new kind of juice. They hire you to calculate the amount of cardboard used for the containers of their new product. Here are the specifications:

- The family size has a capacity of 2 litres.
- The individual size has a capacity of 250 mL.
- The containers are similar.
- The base of the family size is a square with 0.8 dm sides.



In one day, the company produces 3000 units of the family size container and 6000 units of the individual size container.

Determine the quantity of cardboard needed for the production of the containers in one day.

ANSWER KEY

Question 1:

Example of an appropriate method
Measure of the height of small prism

$$A_L = 4c \times h$$
$$120 = 4 \times 5 \times h$$
$$h = 6$$

Volume of small prism (V_s)

$$V_s = A_b \times h$$
$$V_s = 5 \times 5 \times 6$$
$$V_s = 150$$

Volume of large prism (V_l)

$$\left(\frac{6}{12}\right)^3 = \frac{150}{V^1}$$
$$\left(\frac{1}{2}\right)^3 = \frac{150}{V^1}$$
$$V_l = 1200$$

Answer The volume of the larger prism is 1200 cm^3.

Question 2 : B

Question 3:

$\boxed{3}$ Example of an appropriate solution
Volume of the smaller prism

$$V_A = (\text{Area of Base}) \times \text{height}$$
$$V_A = (4\text{ cm} \times 2\text{ cm}) \times (8\text{ cm})$$
$$V_A = 64\text{ cm}^3$$

Ratio of volumes

$$k^3 = \frac{V_B}{V_A}$$
$$k^3 = \frac{1728}{64}$$

Ratio of heights

$$k = \sqrt[3]{k^3}$$
$$k = \sqrt[3]{27}$$
$$k = 3$$

Calculation of the height of the larger prism

$$\frac{h_B}{h_A} = 3$$
$$h_B = 3 \times (8\text{ cm})$$
$$h_B = 24\text{ cm}$$

Answer: The height of the larger prism is **24** cm.

Question 4 :

Work : (example)

Area of park without sidewalk

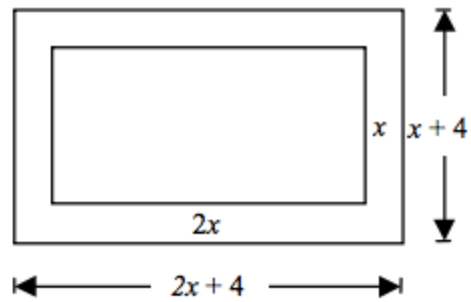
$$x(2x) = 2x^2$$

Area of park with sidewalk

$$(x + 4)\ (2x + 4) = 2x^2 + 12x + 16$$

Increase in area

$$2x^2 + 12x + 16 - 2x^2 = 12x + 16$$



Length of park without sidewalk

$$\begin{aligned} 12x + 16 &= 136 \\ 12x &= 120 \\ x &= 10 \end{aligned}$$

Area of park without sidewalk.

$$x(2x) = 10 \times 20 = 200$$

Result The area of the park without the sidewalk is 200 m^2.

Question 5

Example of an appropriate solution

Height of pool

$$5x + 29 - 9 - 9 = 5x + 11$$

Width of rectangle at each (left and right) edge of pool

$$\frac{8x + 44 - 8x + 20}{2} = 12 \text{ units}$$

Length of corner diagonals

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 9^2 + 12^2 \\ &= 81 + 144 \\ &= 225 \\ c &= \sqrt{225} \\ &= 15 \end{aligned}$$

Perimeter of deck

$$\begin{aligned} 4(15) + 2(8x + 20) + 2(5x + 11) &= 60 + 16x + 40 + 10x + 22 \\ &= 26x + 122 \end{aligned}$$

Answer: The simplified expression for the length of the fence is **26*x* + 122**.

Question 6

Example of an appropriate solution:

$$\frac{\left(\left(a^3 \bullet b^5\right)^8\right)^{\frac{1}{2}} \bullet \left(a^2 \bullet c^3\right)^5 \bullet \left(c^5 \bullet b\right)^{-2} \bullet a^3}{\left(a^3 \bullet c^{-1}\right)^{-5} \bullet \left(b^2\right)^{-7} \bullet \left(\left(b^4 \bullet a^5\right)^{24}\right)^{\frac{1}{3}}} = 1$$

$$\frac{a^{12} \bullet b^{20} \bullet a^{10} \bullet c^{15} \bullet c^{-10} \bullet b^{-2} \bullet a^3}{a^{-15} \bullet c^5 \bullet b^{-14} \bullet b^{32} \bullet a^{40}} = 1$$

$$\frac{a^{25} \bullet b^{18} \bullet c^5}{a^{25} \bullet c^5 \bullet b^{18}} = 1$$

$$\frac{a^{25} \bullet b^{18} \bullet c^5}{a^{25} \bullet b^{18} \bullet c^5} = 1$$

$$1 = 1$$

Question 7

Example of an appropriate solution

Note: 1ℓ = 1 dm^3

∴ Family size = 2ℓ = 2 dm^3

- **The height of the large rectangular prism (family size):**

$$\begin{aligned}\text{Volume} &= \text{Area of square base x height}\\ 2 &= 0.8^2 \text{ x height}\\ \text{height} &= 3.125 \text{ dm}\end{aligned}$$

- **The total area of the large rectangular prism (family size):**

$$\begin{aligned}\text{Total area} &= 2(\text{area of base}) + P_b h\\ &= 2(0.8 \times 0.8) + (4 \times 0.8 \times 3.125)\\ &= 1.28 + 10\\ &= 11.28 \text{ dm}^2\end{aligned}$$

- **Similarity ratio**

The ratio of the volumes is $k^3 = \dfrac{2}{0.25} = 8$

The ratio of the areas is $k^2 = \left(\sqrt[3]{8}\right)^2 = 4$

- **The area of the small rectangular prism (individual size):**

$$\begin{aligned}\text{Area of small container} &= \frac{\text{Area of large container}}{4}\\ &= \frac{11.28}{4}\\ &= 2.82 \text{ dm}^2\end{aligned}$$

- **Amount of cardboard needed:**

$$\begin{aligned}\text{Total Area} &= 3000(11.28) + 6000(2.82)\\ &= 33840 + 16920\\ &= 50760 \ dm^2\end{aligned}$$

Answer: **50 760 dm^2** more cardboard is needed.