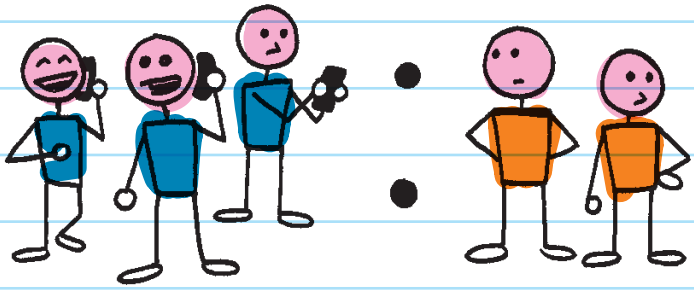
Ratios, Proportions, and Percents

Chapter 15

RATIOS

A **RATIO** is a comparison of two quantities. For example, you might use a ratio to compare the number of students who have cell phones to the number of students who don't have cell phones. A ratio can be written a few different ways.



The ratio 3 to 2 can be written:

$3:2$ or $\frac{3}{2}$ or 3 to 2

Use "a" to represent the first quantity and "b" to represent the second quantity. The ratio a to b can be written:

$a:b$ or $\frac{a}{b}$ or a to b

A fraction can also be a ratio.

EXAMPLES: Five students were asked if they have a cell phone. Four said yes and one said no. What is the ratio of students who do not have cell phones to students who do?

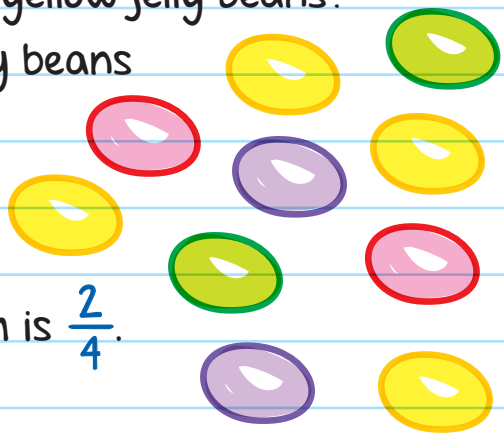
1:4 or $\frac{1}{4}$ or 1 to 4.

(Another way to say this is, "For every 1 student who does not have a cell phone, there are 4 students who do have a cell phone.")

What is the ratio of students who have cell phones to total number of students asked?

4:5 or $\frac{4}{5}$ or 4 to 5.

EXAMPLE: Julio opens a small bag of jelly beans and counts them. He counts 10 total. Among those 10, there are 2 green jelly beans and 4 yellow jelly beans. What is the ratio of green jelly beans to yellow jelly beans? And what is the ratio of green jelly beans to total number of jelly beans?



The ratio of green jelly beans to yellow jelly beans in fraction form is $\frac{2}{4}$. That can be simplified to $\frac{1}{2}$.

So, for every 1 green jelly bean, there are 2 yellow jelly beans.

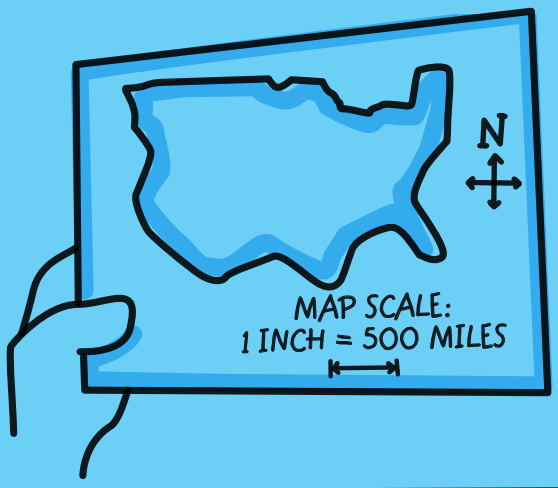
The ratio of green jelly beans to the total amount is $\frac{2}{10}$.

That can be simplified to $\frac{1}{5}$.

So, 1 out of every 5 jelly beans in the bag is green.

Just like you simplify fractions, you can also simplify ratios!

Ratios are often used to make **SCALE DRAWINGS**— a drawing that is similar to an actual object or place but bigger or smaller.



A map shows the ratio of the distance on the map to the distance in the real world.

CHECK YOUR KNOWLEDGE

For 1 through 6, write each ratio as a fraction. Simplify if possible.

1. 2:9
2. 42:52
3. 5 to 30
4. For every 100 apples, 22 apples are rotten.
5. 16 black cars to every 2 red cars
6. 19:37

For 7 through 10, write a ratio in the format of $a:b$ to describe each situation.

7. Of the 27 people surveyed, 14 live in apartment buildings.
8. In the sixth grade, there are 8 girls to every 10 boys.
9. Exactly 84 out of every 100 homes has a computer.
10. Lucinda bought school supplies for class. She bought 8 pens, 12 pencils, and 4 highlighters. What was the ratio of pens to total items?

ANSWERS

CHECK YOUR ANSWERS

1. $\frac{2}{9}$
2. $\frac{21}{26}$
3. $\frac{1}{6}$
4. $\frac{11}{50}$
5. $\frac{8}{1}$
6. $\frac{19}{37}$
7. 14:27
8. 8:10 or 4:5
9. 21:25
10. 8:24 or 1:3

Chapter 16

UNIT RATE AND UNIT PRICE

A **RATE** is a special kind of ratio where the two amounts being compared have different units. For example, you might use rate to compare 3 cups of flour to 2 teaspoons of sugar. The units (cups and teaspoons) are different.

A **UNIT RATE** is a rate that has 1 as its denominator. To find a unit rate, set up a ratio as a fraction and then divide the numerator by the denominator.

EXAMPLE: A car can travel 300 miles on 15 gallons of gasoline. What is the unit rate per gallon of gasoline?

MEANS DIVIDE

$$300 \text{ miles} : 15 \text{ gallons} = \frac{300 \text{ miles}}{15 \text{ gallons}} = 20 \text{ miles per gallon}$$

The unit rate is **20 miles per gallon**.

This means the car can travel 20 miles on 1 gallon of gasoline.

EXAMPLE: An athlete can swim $\frac{1}{2}$ mile every $\frac{1}{3}$ hour. What is the unit rate of the athlete?

In plain English: How many miles per hour can the athlete swim?

$$\frac{1}{2} \text{ mile} : \frac{1}{3} \text{ mile} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$$

$$= 1\frac{1}{2} \text{ miles per hour}$$

When the unit rate describes a price, it is called **UNIT PRICE**. When you're calculating unit price, be sure to put the price in the numerator!

EXAMPLE: Jacob pays $1.60 for 2 bottles of water. What is the unit price of each bottle?

$$\$1.60 : 2 \text{ bottles or } \frac{1.60}{2} = \$0.80$$

The unit price is $0.80 per bottle.

CHECK YOUR KNOWLEDGE

For 1 through 10, find the unit rate or unit price.

1. My mom jogs 30 miles in 5 hours.
2. We swam 100 yards in 2 minutes.
3. Juliette bought 8 ribbons for $1.52.
4. He pumped 54 gallons in 12 minutes.
5. It costs $2,104.50 to purchase 122 soccer balls.
6. A runner sprints $\frac{1}{2}$ of a mile in $\frac{1}{15}$ hour.
7. Linda washes 26 bowls per 4 minutes.
8. Safira spends $42 for 12 gallons of gas.
9. Nathaniel does 240 push-ups in 5 minutes.
10. A team digs 12 holes every 20 hours.

ANSWERS

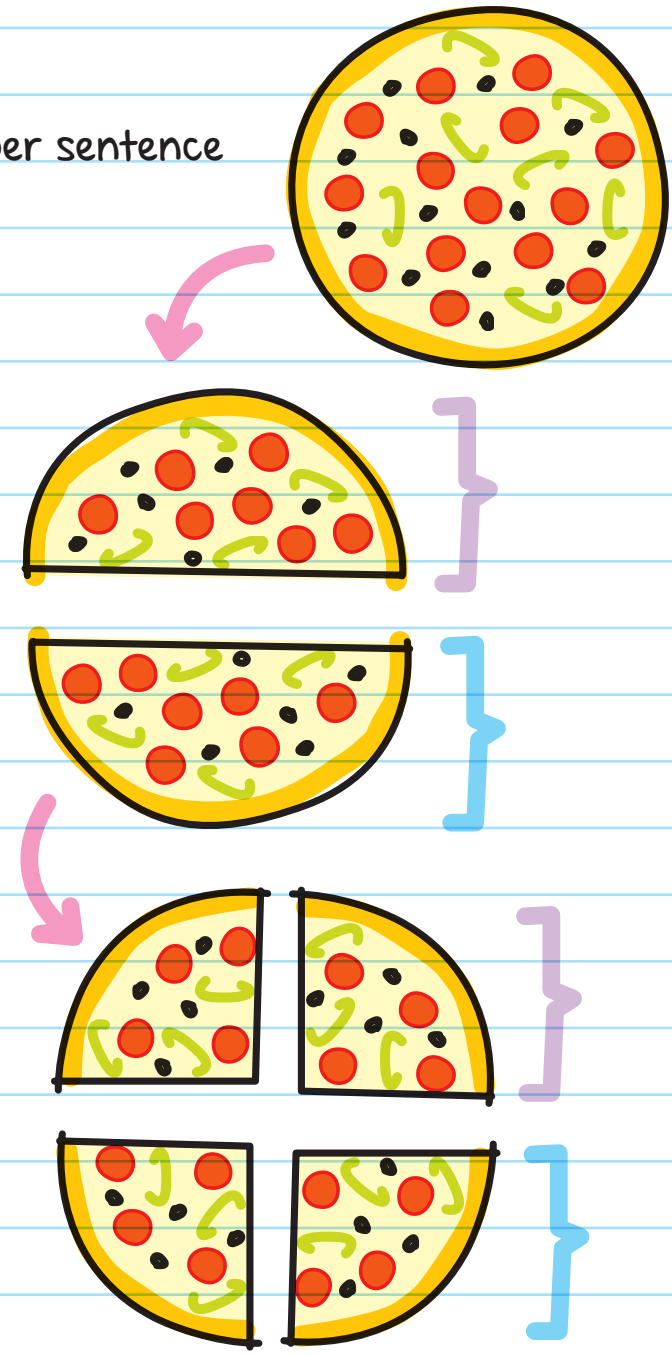
CHECK YOUR ANSWERS

1. 6 miles per hour
2. 50 yards per minute
3. $0.19 per ribbon
4. 4.5 gallons per minute
5. $17.25 per soccer ball
6. $7\frac{1}{2}$ miles per hour
7. 6.5 bowls per minute
8. $3.50 per gallon of gas
9. 48 pushups per minute
10. 0.6 holes per hour

Chapter 17

PROPORTIONS

A **PROPORTION** is a number sentence where two ratios are equal. For example, someone cuts a pizza into 2 equal pieces and eats 1 piece. The ratio of pieces that person ate to the original pieces of pizza is $\frac{1}{2}$. The number $\frac{1}{2}$ is the same ratio as if that person instead cut the pizza into 4 equal pieces and ate 2 pieces.



$$\frac{1}{2} = \frac{2}{4}$$

You can check if two ratios form a proportion by using cross products. To find cross products, set the two ratios next to each other, then multiply diagonally. If both products are equal to each other, then the two ratios are equal and form a proportion.

$\frac{1}{2} \times \frac{2}{4}$

SOMETIMES, TEACHERS ALSO CALL THIS CROSS MULTIPLICATION.

$1 \times 4 = 4$

$2 \times 2 = 4$

$4 = 4$

The cross products are equal, so $\frac{1}{2} = \frac{2}{4}$.

EXAMPLE: Are $\frac{3}{5}$ and $\frac{9}{15}$ proportional?

$\frac{3}{5} \times \frac{9}{15}$

$3 \times 15 = 45$

$9 \times 5 = 45$

$45 = 45$

Two ratios that form a proportion are called **EQUIVALENT FRACTIONS**.

$\frac{3}{5}$ and $\frac{9}{15}$ ARE proportional—their cross products are equal.

You can also use a proportion to **FIND AN UNKNOWN QUANTITY**. For example, you are making lemonade, and the recipe says to use 5 cups of water for every lemon you squeeze. How many cups of water do you need if you have 6 lemons?

First, set up a ratio: $\frac{5 \text{ cups}}{1 \text{ lemon}}$

Second, set up a ratio for what you are trying to figure out. Because you don't know how many cups are required for 6 lemons, use x for the amount of water.

$$\frac{x \text{ cups}}{6 \text{ lemons}}$$

Third, set up a proportion by setting the ratios equal to each other:

$$\frac{5 \text{ cups}}{1 \text{ lemon}} = \frac{x \text{ cups}}{6 \text{ lemons}}$$

NOTICE THAT THE UNITS ACROSS FROM EACH OTHER MATCH.

Last, use cross products to find the missing number!

$1 \cdot x = 5 \times 6$

$1 \cdot x = 30$ (Divide both sides by 1 so you can get x alone.)

$x = 30$

You need 30 cups for 6 lemons!

EXAMPLE: You drive 150 miles in 3 hours. At this rate, how far would you travel in 7 hours?

$$\frac{150 \text{ miles}}{3 \text{ hours}} = \frac{x \text{ miles}}{7 \text{ hours}}$$

$150 \cdot 7 = 3 \cdot x$

$1{,}050 = 3x$ (Divide both sides by 3 so you can get x alone.)

$350 = x$

You'll travel 350 miles in 7 hours.

> Whenever you see "at this rate," set up a proportion!

Sometimes, a proportion stays the same, even in different scenarios. For example, Tim runs $\frac{1}{2}$ a mile, and then he drinks 1 cup of water. If Tim runs 1 mile, he needs 2 cups of water. If Tim runs 1.5 miles, he needs 3 cups of water (and so on). The proportion stays the same, and we multiply by the same number in each scenario (in this case, we multiply by 2). This is known as the **CONSTANT OF PROPORTIONALITY** or the **CONSTANT OF VARIATION** and is closely related to unit rate (or unit price).

EXAMPLE: A recipe requires 6 cups of water for 2 pitchers of fruit punch. The same recipe requires 15 cups of water for 5 pitchers of fruit punch. How many cups of water are required to make 1 pitcher of fruit punch?

We set up a proportion:

$$\frac{6 \text{ cups}}{2 \text{ pitchers}} = \frac{x \text{ cups}}{1 \text{ pitcher}} \quad \text{or} \quad \frac{15 \text{ cups}}{5 \text{ pitchers}} = \frac{x \text{ cups}}{1 \text{ pitcher}}$$

By solving for x in both cases, we find out that the answer is always 3 cups.

We can also see unit rate by using a table. With the data from the table, we can set up a proportion:

EXAMPLE: Daphne often walks laps at the track. The table below describes how much time she walks and how many laps she finishes. How many minutes does Daphne walk per lap?

Total minutes walking	28	42
Total number of laps	4	6

$$\frac{28 \text{ minutes}}{4 \text{ laps}} = \frac{x \text{ minutes}}{1 \text{ lap}} \quad \text{or} \quad \frac{42 \text{ minutes}}{6 \text{ laps}} = \frac{x \text{ minutes}}{1 \text{ lap}}$$

Solving for x, we find out that the answer is 7 minutes.

CHECK YOUR KNOWLEDGE

1. Do the ratios $\frac{3}{4}$ and $\frac{6}{8}$ form a proportion? Show why or why not with cross products.

2. Do the ratios $\frac{4}{9}$ and $\frac{6}{11}$ form a proportion? Show why or why not with cross products.

3. Do the ratios $\frac{4}{5}$ and $\frac{12}{20}$ form a proportion? Show why or why not with cross products.

4. Solve for the unknown: $\frac{3}{15} = \frac{9}{x}$.

5. Solve for the unknown: $\frac{8}{5} = \frac{y}{19}$. Answer in decimal form.

6. Solve for the unknown: $\frac{m}{6.5} = \frac{11}{4}$. Answer in decimal form.

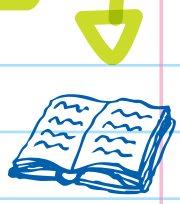
7. In order to make the color pink, a painter mixes 2 cups of white paint with 5 cups of red. If the painter wants to use 4 cups of white paint, how many cups of red paint will she need to make the same color pink?

8. Four cookies cost $7. At this rate, how much will 9 cookies cost?

9. Three bagels cost $2.67. At this rate, how much will 10 bagels cost?

10. It rained 3.75 inches in 15 hours. At this rate, how much will it rain in 35 hours? Answer in decimal form.

CHECK YOUR ANSWERS



1. Yes, because $\frac{3}{4} \times \frac{6}{8}$

$3 \times 8 = 24$

$6 \times 4 = 24$

$24 = 24$

2. No, because $\frac{4}{9} \times \frac{6}{11}$

$4 \times 11 = 44$

$6 \times 9 = 54$

$44 \neq 54$

3. No, because $\frac{4}{5} \times \frac{12}{20}$

$4 \times 20 = 80$

$12 \times 5 = 60$

$80 \neq 60$

4. $x = 45$

5. $y = 30.4$

6. $m = 17.875$

7. 10 cups

8. $15.75

9. $8.90

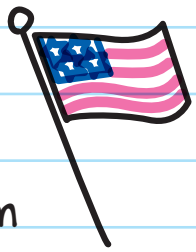
10. 8.75 inches

Chapter 18

CONVERTING MEASUREMENTS

Sometimes, we want to change one type of measurement unit (such as inches) to another unit (such as feet). This is called **CONVERTING MEASUREMENTS**.

STANDARD SYSTEM of MEASUREMENT



In the U.S., we use the **STANDARD SYSTEM** of measurement. Here are some standard system measurements and their equivalent units:

Length

12 inches (in) = 1 foot (ft)

3 feet (ft) = 1 yard (yd)

1,760 yards (yd) = 1 mile (mi)

Weight

1 pound (lb) = 16 ounces (oz)

1 ton (t) = 2,000 pounds (lb)

Capacity

1 tablespoon (tbsp) = 3 teaspoons (tsp)

1 fluid ounce (oz) = 2 tablespoons (tbsp)

1 cup (c) = 8 fluid ounces (oz)

1 pint (pt) = 2 cups (c)

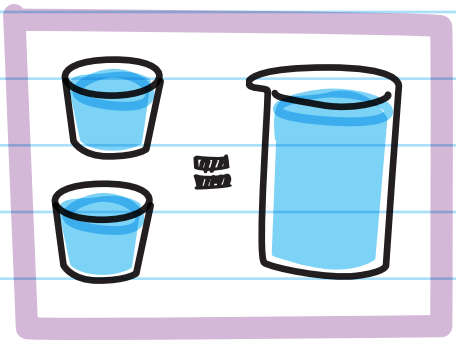
1 quart (qt) = 2 pints (pt)

1 gallon (gal) = 4 quarts (qt)

When converting between measurements, set up a proportion and solve.

EXAMPLE: How many quarts are there in 10 pints?

We already know that 1 quart is the same as 2 pints, so we use this ratio:



$$\frac{x \text{ quarts}}{10 \text{ pints}} = \frac{1 \text{ quart}}{2 \text{ pints}}$$

We cross multiply to find the answer is 5 quarts.

EXAMPLE: How many pints are there in 64 fluid ounces?

We can use ratios and proportions, and repeat this process until we end up with the right units. We already know that there are 8 fluid ounces in 1 cup, so we change from fluid ounces to cups first.

$$\frac{x \text{ cups}}{64 \text{ fluid ounces}} = \frac{1 \text{ cup}}{8 \text{ fluid ounces}}$$

We cross multiply to find the answer is 8 cups.

Next, we change 8 cups to pints.

We already know that there are 2 cups in 1 pint, so we set up another proportion:

$$\frac{x \text{ pints}}{8 \text{ cups}} = \frac{1 \text{ pint}}{2 \text{ cups}}$$

MAKE SURE YOUR UNITS ALWAYS MATCH HORIZONTALLY.

We cross multiply to find the answer is 4 pints.

METRIC SYSTEM of MEASUREMENT

Most other countries use the **METRIC SYSTEM** of measurement. Here are some metric system measurements and their equivalent units:

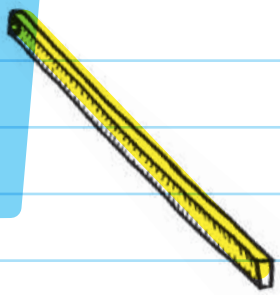
WE ALSO USE THE METRIC SYSTEM IN SCIENCE CLASS!

Length

10 millimeters (mm) = 1 centimeter (cm)

100 centimeters (cm) = 1 meter (m)

1,000 meters (m) = 1 kilometer (km)



Weight

1,000 milligrams (mg) = 1 gram (g)

1,000 grams (g) = 1 kilogram (kg)

When converting between measurements, set up a proportion and solve.

EXAMPLE: How many centimeters are there in 2 kilometers?

We can use ratios and proportions because we already know that there are 1,000 meters in 1 kilometer:

$$\frac{x \text{ meters}}{2 \text{ kilometers}} = \frac{1{,}000 \text{ meters}}{1 \text{ kilometer}}$$

We cross multiply to find the answer is 2,000 meters.

Next, we change 2,000 meters to centimeters.

We already know that there are 100 centimeters in 1 meter, so we set up another proportion:

$$\frac{x \text{ centimeters}}{2{,}000 \text{ meters}} = \frac{100 \text{ centimeters}}{1 \text{ meter}}$$

We cross multiply to find the answer is 200,000 cm.

CONVERTING BETWEEN MEASUREMENT SYSTEMS

Sometimes, we want to change one type of measurement unit (such as inches) to another unit (such as centimeters). When we change units from the standard system to the metric system or vice versa, we are **CONVERTING BETWEEN MEASUREMENT SYSTEMS**.

Here are some of the **COMMON CONVERSIONS OF STANDARD TO METRIC**:

Length

1 inch (in) = 2.54 centimeters (cm)

3.28 feet (ft) = 1 meter (m) (approximately)

1 yard (yd) = 0.9144 meter (m)

1 mile (mi) = 1.61 kilometers (km) (approximately)

Weight

1 ounce (oz) = 28.349 grams (g) (approximately)

1 pound (lb) = 453.592 grams (g) (approximately)

1 pound (lb) = 0.454 kilograms (kg) (approximately)

Capacity

1 fluid ounce (fl oz) = 29.574 milliliters (ml) (approximately)

1 pint (pt) = 473.177 milliliters (ml) (approximately)

1 pint (pt) = 0.473 liters (l) (approximately)

1 gallon (gal) = 3.785 liters (l) (approximately)

When converting between measurement systems, just set up a proportion and solve.

EXAMPLE: How many gallons are in 12 liters?

First, set up a proportion with the unknown quantity as x.

$$\frac{1 \text{ gallon}}{3.785 \text{ liters}} = \frac{x \text{ gallons}}{12 \text{ liters}}$$

Next, use cross products to find the missing number.

$3.785x = 12$

(Divide both sides by 3.785 to isolate x on one side of the equal sign.)

x = approximately 3.17 gallons

So, there are roughly 3 gallons in 12 liters!

CHECK YOUR KNOWLEDGE

For 1 through 8, fill in the blanks.

1. 26 feet = ____ inches

2. ____ gallons = 24 quarts

3. 30 teaspoons = ____ fluid ounces

4. ____ millimeters = 0.08 kilometers

5. 30 centimeters = ____ inches

6. 4.5 miles = ____ feet

7. ____ grams = 36 ounces

8. 5.25 pints = ____ liters

9. While hiking a trail that is 7 miles long, you see a sign that says, "Distance you've traveled: 10,000 feet." How many feet remain in the hike?

10. Mount Everest, on the border of Nepal, is 8,848 meters tall, while Chimborazo in Ecuador is 6,310 meters tall. What is the difference in elevation between the two mountains in feet?

ANSWERS

CHECK YOUR ANSWERS

1. 312
2. 6
3. 5
4. 80,000
5. Approximately 11.81
6. 23,760
7. Approximately 1,020.564
8. Approximately 2.48325
9. 26,960
10. Approximately 8,325.64