

ANALYTIC GEOMETRY

Distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$

how far, length

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between points $A(x_1, y_1)$ and $B(x_2, y_2)$

halfway, middle, divided into equal parts

$$\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}$$

Point of Division: fraction $\frac{a}{b}$ of the way from $A(x_1, y_1)$ to $B(x_2, y_2)$ STARTING FROM A!

part way, division of a line

if given a ratio p:p → convert to fraction $\frac{\text{part}}{\text{whole}}$

$$x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1)$$

EQUATION OF A LINE

Functional form

Equation (rule): $y = ax + b$

Slope, rate of change

y-intercept, initial value

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = y_1 - a(x_1)$$

i.e.: $A(6, 7)$ and $B(15, 25)$

$$1) \frac{25 - 7}{15 - 6} = \frac{18}{9} = 2$$

$$[y = 2x + b]$$

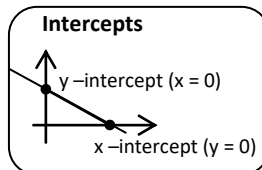
$$2) 7 = 2(6) + b$$

$$7 = 12 + b$$

$$7 - 12 = b$$

$$b = -5$$

$$3) y = 2x - 5$$



General form $ax + by + c = 0$

*convert to Functional form by isolating 'y'

$$\text{Ex. } 3x + 2y - 6 = 0$$

$$2y = -3x + 6 \quad \text{divide by 2}$$

$$y = -1.5x + 3$$

Parallel & Perpendicular LINES

Parallel lines: have the same slope (a)

Find the equation **parallel** to $y = 2x + 8$

And goes through the point (5, 25)

1) Find a: $2 \rightarrow 2$ (same slope)

2) Find b: $25 = 2(5) + b$

$$25 = 10 + b$$

$$25 - 10 = b \rightarrow b = 15$$

3) Write the rule: $y = 2x + 15$

Perpendicular lines: negative reciprocal slopes - flip the fraction **change the sign**

Find the equation **perpendicular** to $y = 3x + 5$

And goes through the point (9, 15)

1) Find a: $3 \rightarrow -1/3$

2) Find b: $15 = -1/3(9) + b$

$$15 = -3 + b$$

$$15 + 3 = b \rightarrow b = 18$$

3) Write the rule: $y = -1/3(x) + 18$

Coincident Lines: SAME slope & b-value

Distinct Lines: DIFFERENT slope & b-value

COMPARISON METHOD

finding a point of intersection with 2 functional form equations

$$y = 2x + 7$$

$$y = -x - 5$$

$$2x + 7 = -x - 5$$

$$2x + x = -5 - 7$$

$$3x = -12$$

$$x = -12 \div 3 = -4$$

$$y = 2(-4) + 7 = -1$$

$$(-4, -1)$$

ELIMINATION METHOD

Finding the value of x and y with 2 general form equations

$$3 \times (2x + 5y = -4)$$

$$2 \times (3x - 2y = 13)$$

$$- \quad 6x + 15y = -12$$

$$- \quad 6x - 4y = 26$$

$$19y = -38$$

$$y = -38 \div 19 = -2$$

$$2x + 5(-2) = -4$$

$$2x = -4 + 10$$

$$x = 6 \div 2 = 3$$

$$(3, -2)$$

Substitution Method

Finding the solution or point of intersection when One equation has x or y isolated

$$y = 4x + 5$$

Sub (4x+5) into (y)

$$3x + 2y = 43$$

$$3x + 2(4x + 5) = 43$$

$$3x + 8x + 10 = 43$$

$$11x + 10 = 43$$

$$11x = 43 - 10$$

$$11x = 33 \div 11$$

$$x = 3$$

$$y = 4(3) + 5$$

$$y = 12 + 5 = 17$$

Solution Set (3, 17)

Strategies to Find a Missing Point (x,y)

Midpoint

If it's halfway, in the middle, in the center

Point of Division

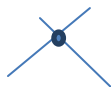
if fraction (leave a fraction)

if ratio (transform into a fraction)

part:part → $\frac{\text{part}}{\text{whole}}$

Systems of Linear Relation

- Comparison
- Substitution
- Elimination



To find the intersection or corner where lines meet.

Note: you may have to find the equation of a line

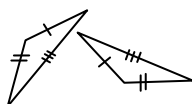
$$y = ax + b$$

Find a (slope) and b (y-intercept)

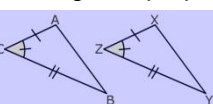
Congruent Triangles

Exactly the same

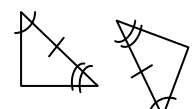
Side Side Side (SSS)



Side Angle Side (SAS)



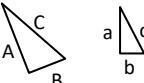
Angle Side Angle (ASA)



Similar Triangles

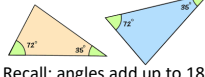
Larger/smaller by scale factor k

Side Side Side (SSS)



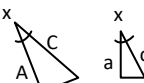
$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c} = k$$

Angle Angle (AA)



Recall: angles add up to 180°

Side Angle Side (SAS)



$$\frac{A}{a} = \frac{C}{c} = k \quad < x = < x$$

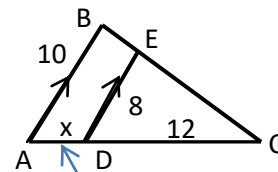
MISSING MEASURES IN TRIANGLES

Similar triangles: Draw two triangles, set up ratio and cross multiply

$$\frac{AB}{DE} = \frac{AC}{DC}$$

$$\frac{10}{8} = \frac{AC}{12}$$

$$AC = 15$$



$$x = 15 - 12$$

$$x = 3$$

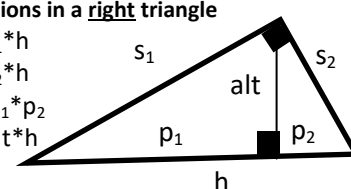
Metric relations in a right triangle

$$S_1^2 = p_1 \cdot h$$

$$S_2^2 = p_2 \cdot h$$

$$\text{alt}^2 = p_1 \cdot p_2$$

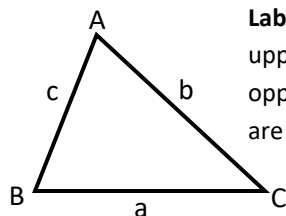
$$S_1 \cdot S_2 = \text{alt} \cdot h$$



Don't forget **Pythagoras!**

$$a^2 + b^2 = c^2$$

SINE LAW: for all triangles



Labels: angles are upper case (A) sides opposite to angles are lower case (a)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

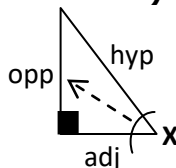
→ Fill in formula, remove extra ratio, cross multiply

remember

- if your unknown is an **angle**, use \sin^{-1}
- if your angle **SHOULD** be obtuse ($> 90^\circ$)... 180° - answer

TRIGONOMETRY: for RIGHT triangles!

(soh cah toa)



To determine measure of angle X, use inverse function

$$\sin^{-1} \quad \cos^{-1} \quad \tan^{-1}$$

remember

- your calculator must be in DEGREE mode
- keep 4 decimal places in your ratio

$$\sin X = \frac{O}{h}$$

$$\cos X = \frac{a}{h}$$

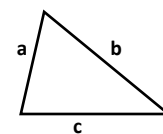
$$\tan X = \frac{o}{a}$$

AREA OF TRIANGLES

$$1. \frac{\text{base} \times \text{height}}{2}$$

*use this when the base is perpendicular to the height (look for right angle!)

2. **HERO'S formula:** use when you know all 3 sides



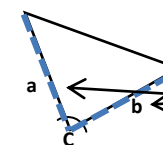
$$\text{calculate semi-perimeter } p = \frac{a+b+c}{2}$$

$$\text{Area} = \sqrt{p(p-a)(p-b)(p-c)}$$

BEDMAS

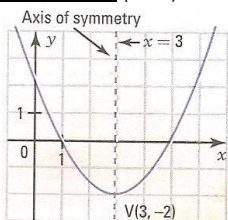
3. **Trigonometric (sandwich) formula:** use when you know 2 sides and one angle (you might need SINE LAW)

$$\text{Area} = \frac{a \times \sin C \times b}{2}$$



given sides **a** and **b**
"sandwich" the given angle **C**

PROPERTIES of Functions (study of a function)



Domain (All x-values)

How far left, How far right : $]-\infty, +\infty[$ or \mathbb{R}

Range (All y-values)

How far down, How far up : $[-2, +\infty[$

Intercepts (where the curve touches an axis)

Zeros or x-intercepts (x,0) : $\{1, 5\}$

Initial Value or y-intercept (0,y) : $\{2\}$

Variation (x-values)

Increasing (going up) : $[3, +\infty[$

Decreasing (going down) : $]-\infty, 3]$

Sign (x-values)

Positive (curve is above x-axis): $]-\infty, 1] \cup [5, +\infty[$

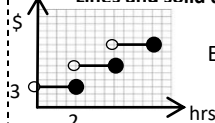
Negative (curve is below x-axis): $[1, 5]$

Extrema (y-values) → if infinite, no extreme

Max = None **Min** = -2

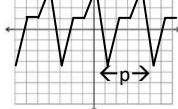
STEP FUNCTION – open circles are **not** included

Lines and solid dots are included



Ex. Pay \$3 for every 2 hours in a parking lot

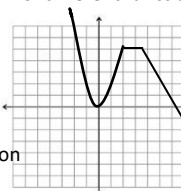
PERIODIC FUNCTION – repeating pattern / wave
Period (**p**) = the length of one repeating pattern



To find a y-value given pattern length, **SUBTRACT** period from given x-value until readable on graph.

PIECEWISE FUNCTION – mix of different functions

$$f(x) = \begin{cases} x^2 & x \leq 5 \\ 25 & 5 < x \leq 10 \\ -2x + 45 & x > 10 \end{cases}$$



- Choose rule that fits x-value
- Plug **given** value into equation
- Solve for **missing** value

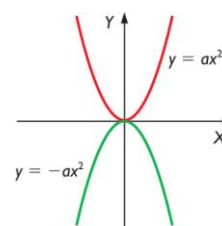
QUADRATIC FUNCTION

(second-degree polynomial function)

$$f(x) = ax^2$$

F(x) = y value

f(#) means replace the x with # solve for y



0 < a < 1 wider (a is a fraction or decimal)
a > 1 thinner

To determine "a", plug in one point on curve

Ex. Point (3, 27) → $y = ax^2$

$$27 = a(3)^2$$

****BEDMAS**

$$27 = a(9) \quad \text{divide by 9}$$

$$3 = a \rightarrow y = 3x^2$$